Course Title: AP CALCULUS (BC) Board Approval Date: Credit / Hours: 1 Reviewed Annually

Course Description:

AP Calculus BC is roughly equivalent to both first and second semester college calculus courses and extends the content learned in AB to different types of equations and introduces the topic of sequences and series. This course covers topics in differential and integral calculus, including concepts and skills of limits, derivatives, definite integrals, the Fundamental Theorem of Calculus, and series. You will learn how to approach calculus concepts and problems when they are represented graphically, numerically, analytically, and verbally, and to make connections amongst these representations.

You will also learn how to use technology to help solve problems, experiment, interpret results, and support conclusions.

*Students will need a TI-89 graphing calculator for this course.

Learning Activities / Modes of Assessment:

Large Group instruction Checklists/Teacher Observation. Projects with Rubrics Homework Assignments Tests and Quizzes Small Group Work Computer simulations

Instructional Resources:

AP*Edition Calculus of a Single Variable(Brooks/Cole, Cengage Learning 2010)

Cou	Course: AP Calculus (BC)			
Course Unit (Topic) (Days/Periods)		Length of Instruction		
1.	Unit #1 Review of AP Calculus (AB)	10 days		
2.	Unit #2 Differential Equations	15 days		
3.	Unit #3 Applications of Integration	12 days		
4.	Unit #4 Advanced Integration Techniques	10 days		
5.	Unit #5 Indeterminate Forms, Improper Integrals	6 days		
6.	Unit #6 Sequences and Series.	25 days		
7.	Unit #7 Taylor Polynomials and Power Series	20 days		
8.	Unit #8 Parametric Equations	15 days		
9.	Unit #9 Polar Curves	<u>15 days</u>		
Tota	l Days	128 days		

Know:

Students will be expected to maintain their knowledge of the topics presented to them throughout AP Calculus (AB).

Understand:

Derivatives can be used to calculate instantaneous rates of change.

Integrals can be used to calculate accumulations over time.

Do:

Students were taught how to find derivatives and integrals throughout AP Calculus (AB). Students will be expected to maintain their skills in calculating derivatives and integrals and use these skills to solve rigorous problems of the type presented on the Advanced Placement Calculus Exam.

BC Calculus Unit 1: Review of AP Calculus (AB)

Unit Essential Question:

What are the key concepts students need to know from AB Calculus?

<u>Concept:</u>	<u>Concept:</u>	<u>Concept:</u>	<u>Concept:</u>
Limits of Functions	Derivatives and their applications	Integrals and their applications	
-	-	-	-
Lesson Essential	Lesson Essential	Lesson Essential	Lesson Essential
<u>Question/s:</u>	<u>Question/s:</u>	<u>Question/s:</u>	<u>Question/s:</u>
Refer to AB Calculus maps	Refer to AB Calculus	Refer to AB Calculus	
	maps	indp3	
-	-	-	-
<u>Vocabulary:</u>	<u>Vocabulary:</u>	<u>Vocabulary:</u>	<u>Vocabulary:</u>
Refer to AB Calculus maps	Refer to AB Calculus maps	Refer to AB Calculus maps	

Unit 2: Differential Equations Time Frame: 15 days

AP Calculus (BC) KUD Text: 6.1, 6.2, 6.3 AP Calculus Standards: APC.8, APC.11, APC.14 (See attached document)

Know:	Understand:	Do:
Differential Equation	Real world phenomena can	Students will be able to:
General Solution	be modeled by using differential equations. Being	Use initial conditions to find
Particular Solution	able to manipulate, visualize,	differential equations
Initial Conditions	can help us to better	Draw slope fields to visualize
Slope Field	understand these	solutions to differential
Euler's Method	pnenomena.	equations
Separation of Variables		Use Euler's Method to approximate solutions to
Proportionality Constant		differential equations
Exponential Growth		Use separation of variables
Exponential Decay		equations
Newton's Law of Cooling		Use exponential functions to model growth and decay in applied problems
Separable Differential Equation		
Logistic Differential Equation		Recognize when separation
Carrying Capacity		of variables is a viable solution technique
		Solve logistic differential equations

BC Calculus Unit 2: Differential Equations

Unit Essential Question:

How do we visualize, solve, and approximate the solutions to differential equations?

<u>Concept:</u> Visualizing the solution to a differential equation using a slope field	<u>Concept:</u> Solving separable differential equations	<u>Concept:</u> Approximating the solution to a differential equation	<u>Concept:</u>
Lesson Essential	Lesson Essential	Lesson Essential	Lesson Essential
Question/s:	<u>Question/s:</u>	<u>Question/s:</u>	Question/s:
How can a slope field be used to visualize the solution to a differential equation? What is the technique used to draw the particular solution to a differential equation on a slope field?	How is separation of variables used to solve a differential equation? What are the general and particular solutions to a differential equation?	How is Euler's Method used to approximate the solution to a differential equation?	

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<u>Vocabulary:</u>	<u>Vocabulary:</u>	<u>Vocabulary:</u>	<u>Vocabulary:</u>
Differential equation, slope field	General solution, particular solution, initial conditions, separation of variables, proportionality constant, exponential growth/decay, Newton's Law of Cooling, separable differential equation, logistic differential equation, carrying capacity	Euler's Method	
	I I		

AP Calculus (BC) KUD Text: 7.1, 7.2, 7.3, 7.4 AP Calculus Standards: APC.13, APC.15 (See attached document)

Know:	Understand:	Do:
Area between two curves	The definite integral can be used to calculate arc lengths, areas, and volumes for irregular curves, shapes, and solids.	Students will be able to:
Representative Rectangle		Describe integration as an accumulation process
Axis of Revolution Disk Method		Calculate the area between two curves using both vertical and horizontal
Representative Disk		representative rectangles
Washer Method Representative Washer		Compute the volume of a solid of revolution using the
		disk, washer, and shell methods
Solid with a known cross section		Compute the volume of a
Shell Method		section
Representative Shell		Decide when it is best to use
Arc Length		the disk/washer method as opposed to the shell method
		Calculate the length of a given curve using integration

BC Calculus Unit 3: Applications of Integration

Unit Essential Question:

What are the different situations to which an integral can be applied?

<u>Concept:</u>	<u>Concept:</u>	<u>Concept:</u>	<u>Concept:</u>
Calculating the area of an	Calculating the volume	Calculating the volume	Calculating the length of a
irregular shape	of a solid of revolution	of a solid with a known	curve
		cross-section	
-	-	-	-
Lesson Essential	Lesson Essential	Lesson Essential	Lesson Essential
<u>Question/s:</u>	<u>Question/s:</u>	<u>Question/s:</u>	Question/s:
How can an integral be	How can an integral be	How can an integral be	How can an integral be
used to calculate the area	used to calculate the	used to calculate the	used to calculate the length
of an irregular shape?	volume of a solid of	volume of a solid with	of a piece of a given curve?
	revolution?	a known cross-section?	
-	-	-	-
<u>Vocabulary:</u>	Vocabulary:	<u>Vocabulary:</u>	<u>Vocabulary:</u>
Area between two	Solid of revolution, axis	Solid with a known cross-	Arc length
curves, representative	of revolution, disk	section	
rectangle	method, representative		
	disk, washer method,		
	representative washer,		
	shell method,		
	representative shell		

Unit 4: Advanced Integration Techniques Time Frame: 10 days

AP Calculus (BC) KUD Unit Text: 8.1, 8.2, 8.5 AP Calculus Standards: APC.13, APC.15 (See attached document)

Know:	Understand:	Do:
Basic integration rules	Many indefinite and definite	Students will be able to:
U-Substitution	integrals require the use of advanced integration	Compute integrals using the techniques taught to them
Expansion	techniques. Being able to	throughout AP Calculus (AB)
Separating the numerator	expands the range of	Calculate an antiderivative
Completing the square	problems for which students	using the technique of
Synthetic Division	given an accumulation	
Long Division	challenge.	Calculate an antiderivative using the tabular method
Addition of zero		Determine when an integral
Trigonometric identities		may be done more easily using the tabular method
Pythagorean Conjugate		Understand the concent of
Integration by Parts		partial fraction
Tabular Method		decomposition
Partial fraction decomposition		Use the method of partial fractions to compute integrals with distinct linear
Method of Partial Fractions		factors
Distinct Linear Factors		

BC Calculus Unit 4: Advanced Integration Techniques

Unit Essential Question:

What are the advanced techniques of integration that students will be required to know for the BC Calculus exam?

<u>Concept:</u>	<u>Concept:</u>	<u>Concept:</u>	<u>Concept:</u>
Basic Integration Concepts	Integration by Parts	Integration using	
		Partial Fractions	
	-		-
Lesson Essential	Lesson Essential	<u>Lesson Essential</u>	<u>Lesson Essential</u>
<u>Question/s:</u>	Question/s:	<u>Question/s:</u>	<u>Question/s:</u>
What are the basic	When can integration	When can partial	
concepts that can be used	by parts be used to	fractions be used to	
to calculate common	calculate an	calculate an	
antiderivatives?	antiderivative?	antiderivative?	
	•		-
Vocabulary:	Vocabulary:	<u>Vocabulary:</u>	<u>Vocabulary:</u>
Basic integration rules, u- substitution, expansion, separating the numerator, completing the square, synthetic division, long division,	Integration by parts, tabular method	Partial fraction decomposition, method of partial fractions, distinct linear factors	

AP Calculus (BC) KUD Unit 5: Indeterminate Forms, Improper Integrals Text: 8.7, 8.8 Time Frame: 6 days AP Calculus Standards: APC.2, APC.8, APC.9 (See attached document)

Know:	Understand:	Do:
Indeterminate Forms	There are several	Students will be able to:
L'Hopital's Rule	indeterminate forms which will cause a limit to be	Recognize limits that
Extended Mean Value Theorem	difficult to compute. These limits will arise when	forms
Improper Integral	attempting to calculate an improper integral. These	Apply L'Hopital's Rule (sometimes multiple times)
Infinite Discontinuity	types of integrals must be	to evaluate a limit
Convergent	techniques learned in this	Manipulate certain types of
Divergent	unit.	limits to get them in a form for which L'Hopital's Rule will
Gabriel's Horn		succeed

Evaluate an improper integral that has an infinite limit of integration

Evaluate an improper integral that has an infinite discontinuity

Calculate the volume and surface area of Gabriel's Horn

Recognize that improper integrals can often lead to non-intuitive results

BC Calculus Unit 5: Indeterminate Forms and Improper Integrals

Length of instruction: 6 Days

Unit Essential Question:

What are the various indeterminate forms, and how can an improper integral be calculated?

<u>Concept:</u> Calculating a limit when an indeterminate form is encountered	<u>Concept:</u> Evaluating an improper integral with an infinite limit of integration	<u>Concept:</u> Evaluating an improper integral with an interior discontinuity	<u>Concept:</u>
	-	-	-
Lesson Essential Question/s: What are the various indeterminate forms? How can a limit be found when an indeterminate form is encountered?	Lesson Essential Question/s: What technique is used to calculate an improper integral with an infinite limit of integration?	<u>Lesson Essential</u> <u>Question/s:</u> What technique is used to calculate an improper integral with an interior discontinuity?	<u>Lesson Essential</u> <u>Question/s:</u>
	-	-	-
<u>Vocabulary:</u> Indeterminate forms, L'Hopital's Rule, Extended Mean Value Theorem	Vocabulary: Improper integral, infinite discontinuity, convergent, divergent, Gabriel's Horn	Vocabulary: Improper integral, interior discontinuity, convergent, divergent	<u>Vocabulary:</u>

AP Calculus (BC) KUD Text: 9.1, 9.2, 9.3, 9.4, 9.5, 9.6 AP Calculus Standards: APC.16 (See attached document)

Unit 6: Sequences and Series Time Frame: 25 days

Know:	Understand:	Do:
Infinite sequence	Calculus is the study of the	Students will be able to:
Nth term	infinite and the infinitesimal.	Determine whether a
Convergent sequence	infinite sequences and series	sequence converges or diverges
Divergent sequence	discipline. Being able to	Write a formula for the nth
Factorial	divergence of a sequence or a series allows a student to	term of a sequence
Absolute Value Theorem		Use properties of monotonic and bounded sequences
Monotonic sequence	further explore the complexities of Calculus.	Understand what it means
Bounded sequence		for an infinite series to be
Infinite series		convergent
Partial sum		Calculate the sums of
Convergent series		series
Divergent series		Use the nth term test
Sum of a series		Use the integral test and p- series test to determine convergence/divergence
Telescoping series		
Geometric series		Use the direct and limit
Nth term test for divergence		comparison tests to determine convergence/divergence
Fractal		
Integral test		Use the alternating series
p-series test		test to determine convergence/divergence
Harmonic series		
Direct Comparison test		Use the alternating series remainder to approximate
Limit Comparison test		the sum of a series
Alternating Series test		Classify a series as
Alternating Series remainder		absolutely/conditionally convergent
Absolute convergence		Use the ratio and root tests to determine
Conditional convergence		
Ratio/Root tests		convergence/ divergence

BC Calculus Unit 6: Sequences and Series

Unit Essential Question:

How can we determine whether a sequence or series converges or diverges?

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<u>Concept:</u>	<u>Concept:</u>	<u>Concept:</u>	<u>Concept:</u>
How can the	How can we calculate	How can	What is the difference
convergence/divergence of	or approximate the	convergence/divergence	between absolute and
a sequence be	sums of various types	of a series be	conditional convergence?
determined?	of series?	determined?	
-	-	-	-
Lesson Essential	Lesson Essential	Lesson Essential	Lesson Essential
<u>Question/s:</u>	<u>Question/s:</u>	<u>Question/s:</u>	<u>Question/s:</u>
How can the nth term	What is a telescoping	What are the various	What does it mean when a
expression be used to	series, and how can we	tests that can be used to	series is absolutely
determine	calculate its value?	determine	convergent?
convergence/divergence of	What is a geometric	convergence/divergence	What does it mean when a
a sequence?	series, and how can we	of a series?	series is conditionally
What does it mean when a	calculate its value?		convergent?
sequence is monotonic and			
bounded?	How can the		
	alternating series		
	remainder be used to		
	approximate the sum		
	of a series?		
-	-	-	-
<u>Vocabulary:</u>	<u>Vocabulary:</u>	<u>Vocabulary:</u>	<u>Vocabulary:</u>
Infinite sequence, nth	Infinite series, partial	Nth term test for	Absolute convergence,
term, convergent	sum, convergent series,	divergence, integral test,	conditional convergence
sequence, divergent	divergent series, sum of a	p-series test, harmonic	
sequence, factorial,	series, telescoping series,	series, direct comparison	
absolute value theorem,	geometric series, fractal,	test, limit comparison	
monotonic sequence,	alternating series	test, alternating series	
bounded sequence	remainder	test, ratio/root tests	

AP Calculus (BC) KUD Unit 7: Tay Text: 9.7, 9.8, 9.9, 9.10 AP Calculus Standards: APC.9, APC.13, APC.17 (See attached document)

Know:	Understand:	Do:
Polynomial approximation of	Functions can often be	Students
a function Centered at c	approximated by using simpler polynomial functions. At times, these	Find poly approxim
Taylor polynomial	polynomials can be extended	elementa
Maclaurin polynomial	to the infinite, creating an exact model of the given	Find Tayl polynom
Remainder of a	function. Modern calculators	of eleme
Taylor/Maclaurin polynomial	and computers extensively use these ideas. Because of	Use the r
Lagrange form of the remainder	this, any student who wishes to move further into a	Taylor po determir
Power series	variety of high tech fields	approxim
Radius of convergence	should have a solid knowledge of the material in	Understa a power
Interval of convergence	this unit.	Find the
Differentiating the power series		of conve series
Integrating the power series		Determir
Geometric power series		series
Taylor series		Different
Maclaurin series		power se
Power series formulas for elementary functions		Find a ge series tha function
		Construe

Students will be able to:

Find polynomial approximations of elementary functions

Find Taylor and Maclaurin polynomial approximations of elementary functions

Use the remainder of the Taylor polynomial to determine the accuracy of an approximation

Understand the definition of a power series

Find the radius and interval of convergence of a power series

Determine the endpoint convergence of a power series

Differentiate and integrate a power series

Find a geometric power series that represents a function

Construct a power series using series operations

Find an infinite Taylor/Maclaurin series for a given function

Use a basic list of Taylor/Maclaurin series to find other Taylor/Maclaurin series

BC Calculus Unit 7: Taylor Polynomials and Power Series

Unit Essential Question:

How can Taylor Polynomials and Power Series be used to model various types of functions?

<u>Concept:</u>	<u>Concept:</u>	Concept:	Concept:
Taylor polynomials	Determining the accuracy of a Taylor polynomial	Power series	Taylor series
-	-	-	-
Lesson Essential Question/s: How can a Taylor polynomial be developed for a given function?	Lesson Essential Question/s: How can the remainder of a Taylor polynomial be calculated in order to determine its accuracy?	Lesson Essential Question/s: How can a power series be developed for certain functions?	<u>Lesson Essential</u> <u>Question/s:</u> How can the concept of a Taylor polynomial be extended to a Taylor series?
	-	-	-
<u>Vocabulary:</u>	Vocabulary:	<u>Vocabulary:</u>	<u>Vocabulary:</u>
Polynomial approximation of a function, centered at c, Taylor polynomial, Maclaurin polynomial	Remainder of a Taylor/Maclaurin polynomial, Lagrange form of the remainder	Power series, radius of convergence, interval of convergence, differentiating the power series, integrating the power series, geometric power series	Taylor series, Maclaurin series, power series formulas for elementary functions

AP Calculus (BC) KUD Text: 10.2, 10.3 AP Calculus Standards: APC.8, APC.9, APC.15 (See attached document)

Know:

Parameter

Parametric equation

Plane curve

Orientation of the curve

Eliminating the parameter

Brachistochrone

Parametric form of the derivative

Parametric form of higherorder derivatives

Arc Length in parametric form

Understand:

It is possible to use three variables to represent a curve in the coordinate plane. These are called parametric curves, and allow us to expand the types of graphs we can create and use. Students will learn how to work with the derivatives of these functions in much the same way they did for elementary functions.

Do:

Students will be able to:

Sketch the graph of a set of parametric equations

Eliminate the parameter in a set of parametric equations

Find a set of parametric equations to represent a given curve

Understand the Brachistochrone problem, which is a classic Calculus challenge

Calculate the slope of a tangent line to a given set of parametric equations

Calculate higher-order derivatives for a set of parametric equations

Calculate the length of a parametric curve on a given domain

BC Calculus Unit 8: Parametric Equations

Unit Essential Question:

What are parametric equations, and how can we use them in calculus?

<u>Concept:</u>	<u>Concept:</u>	<u>Concept:</u>	<u>Concept:</u>
Graphing parametric	Eliminating the	The calculus of	Vector Valued Functions
equations	parameter	parametric equations	
-	-	-	-
<u>Lesson Essential</u>	Lesson Essential	<u>Lesson Essential</u>	<u>Lesson Essential</u>
<u>Question/s:</u>	<u>Question/s:</u>	<u>Question/s:</u>	<u>Question/s:</u>
How can a parametric equation be graphed by hand? How can a parametric equation be graphed using technology?	How can we convert a parametric equation to its rectangular form?	How can we calculate the first and second derivatives of parametric equations? How can we calculate the length of the arc of a parametric curve?	How can we find the velocity and acceleration vectors? How can we calculate the speed of an object moving along a parametric path? How can we calculate the total distance traveled by an object moving along a parametric path?
Vocabulary:	Vocabulary:	Vocabulary:	Vocabulary:
Parameter, parametric equation, plane curve, orientation of the curve, brachistochrone	Eliminating the parameter	Parametric form of the derivative, parametric form of higher-order derivatives, arc length in parametric form	Velocity vector, acceleration vector, speed in parametric form, total distance traveled in parametric form

AP Calculus (BC) KUD Text: 10.4, 10.5 AP Calculus Standards: APC.8, APC.9, APC.15 (See attached document)

Know:

Polar coordinate system

Polar-to-rectangular conversion

Rectangular-to-polar conversion

Slope in polar form

Limacon

Rose

Lemniscate

Area in polar coordinates

Area of a region between two polar curves

Understand:

By using a new coordinate system, students will be able to graph, study, and work with entirely new classes of functions. Polar coordinates allow us to solve old problems in new ways and work with functions that were not previously possible. We can extend our use of the derivative and integral to polar coordinates to study those Calculus concepts.

Do:

Students will be able to:

Understand the polar coordinate system

Convert from polar to rectangular equations

Convert from rectangular to polar equations

Sketch the graph of a given polar curve

Identify several types of special polar curves

Graph a variety of limacons, roses, and lemniscates

Calculate the slope of a tangent line to a polar curve

Find horizontal and vertical tangent lines to polar curves

Calculate the area of a region bounded by a polar curve

Find the points of intersection of two polar curves

Calculate the area between two polar curves

BC Calculus Unit 9: Polar Curves

Unit Essential Question:

What are the calculus concepts we can apply to polar curves?

<u>Concept:</u>	<u>Concept:</u>	<u>Concept:</u>	<u>Concept:</u>
Graphing polar curves	Converting from polar to rectangular and back	Applying calculus to polar curves	
•	-	-	-
Lesson Essential	Lesson Essential	Lesson Essential	Lesson Essential
Question/s:	Question/s:	Question/s:	Question/s:
How can polar curves be graphed by hand? How can polar curves be graphed using technology?	How can one convert from polar form to rectangular form? How can one convert from rectangular form to polar form?	How can one calculate the derivative of a polar curve? How can the area of a region between two polar curves be calculated?	

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<u>Vocabulary:</u>	<u>Vocabulary:</u>	<u>Vocabulary:</u>	<u>Vocabulary:</u>
Polar coordinate system, limacon, rose, lemniscate	Polar-to-rectangular conversion, rectangular- to-polar conversion	Slope in polar form, area in polar coordinates, area of a region between two polar curves	